Clues for person F

11

Calculate the area between the curve and the x-axis.

The y-intercept of the curve is (0, 3).

12

Let y = price of each toy.

To cover costs y should be $\dfrac{\text{total expenditure}}{\text{number of toys to sell}}$.

13

The 20 staff supervisors come for free.

The band costs $400 to hire for the night.

14

Every 20 minutes three more people than the number already present arrive and join the queue.

15

The number of insects Joe found followed a pattern.

What is the total number of insects found in the search?

16

Ellen had never been to Noonga before.

The rain falls in the African jungle 80% of the time.

17

There are 12 girls in the class.

The principals' name is Mrs Leeming.

18

The sailor took 10 cans of food with him.

What is the probability he had two tins of baked beans for dinner?

19

The speeds of the cars follow a normal distribution.

The weather was wet and windy.

20

1200 people were surveyed. Each counted their pulse for one minute.

Completing the square (1)

Find the correct order for each set of steps.

A

$x^2 - 2x = 3$

B

$(x - 1)^2 = 4$

C

By completing the square, solve

$$x^2 - 2x - 3 = 0$$

D

$x = -1, 3$

E

$x^2 - 2x + 1 = 4$

F

$x - 1 = \pm 2$

G

Half of -2 is -1

$(-1)^2$ is 1

Completing the square (2)

A

$(x + 3)^2 = 16$

B

$x^2 + 6x = 7$

C

$x = -7, 1$

D

Half of 6 is 3

3^2 is 9

E

Solve by completing the square

$$x^2 + 6x - 7 = 0$$

F

$x + 3 = \pm 4$

G

$x^2 + 6x + 9 = 16$

Locating turning points (1)

Find the correct order for each set of steps.

A

At the turning points the gradient is equal to zero.

$3x^2 - 8x - 3 = 0$

B

Find the turning points of the function

$$f(x) = x^3 - 4x^2 - 3x + 2$$

and state their nature.

C

The double derivative gives the nature of the turning points.

$f''(x) < 0$: maximum

$f''(x) > 0$: minimum

$f''(x) = 6x - 8$

D

$f(-0.\dot{3}) = 2.52$

$f(3) = -16$

E

$f'(x) = 3x^2 - 8x - 3$

F

$f''(-0.\dot{3}) = -10$

so $(-0.\dot{3}, 2.52)$ is a local maximum

$f''(3) = 10$

so $(3, -16)$ is a local minimum

G

$(3x + 1)(x - 3) = 0$

$x = -0.\dot{3}, 3$

Locating turning points (2)

A

$(3x - 5)(x + 2) = 0$

$x = -2, 1.\dot{6}$

B

The double derivative gives the nature of the turning points.

$f''(x) < 0$: maximum

$f''(x) > 0$: minimum

$f''(x) = 6x + 1$

C

$f''(-2) = -11$

so $(-2, 19)$ is a local maximum.

$f''(1.\dot{6}) = 11$

so $(1.\dot{6}, -5.65)$ is a local minimum.

D

$f'(x) = 3x^2 + x - 10$

E

$f(-2) = 19$

$f(1.\dot{6}) = -5.65$

F

Determine the nature and location of the turning points of

$$f(x) = x^3 + \frac{x^2}{2} - 10x + 5$$

G

At the turning points of a function the gradient is equal to zero.

$3x^2 + x - 10 = 0$

Standard deviation proof

Rearranging equations (1)

Find the correct order for each set of steps.

Standard deviation proof

A

$$= \sqrt{\frac{\sum x_i^2 - \sum 2x_i\bar{x} + \sum \bar{x}^2}{n}}$$

B

$$= \sqrt{\frac{\sum x_i^2 - 2n\bar{x}^2 + n\bar{x}^2}{n}}$$

C

$$= \sqrt{\frac{\sum (x_i^2 - 2x_i\bar{x} + \bar{x}^2)}{n}}$$

D

Prove

$$\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n}}$$

E

$$= \sqrt{\frac{\sum x_i^2 - 2\bar{x}\sum x_i + n\bar{x}^2}{n}}$$

F

$$= \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n}}$$

G

$$\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

Rearranging equations (1)

A

$$\sqrt{\frac{l}{g}} = \frac{T}{2\pi}$$

B

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

C

$$l = \frac{T^2 g}{4\pi^2}$$

D

$$\frac{l}{g} = \frac{T^2}{4\pi^2}$$

E

Make l the subject of the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Rearranging equations (2)

Find the correct order for each set of steps.

A

Make x the subject of the formula

$$\frac{1}{x^2} - \frac{1}{y^2} = \frac{1}{a^2}$$

B

$$x^2 = \frac{a^2 y^2}{a^2 + y^2}$$

C

$$x = \frac{ay}{\sqrt{a^2 + y^2}}$$

D

$$\frac{1}{x^2} = \frac{a^2 + y^2}{a^2 y^2}$$

E

$$\frac{1}{x^2} = \frac{1}{a^2} + \frac{1}{y^2}$$

Rearranging equations (3)

A

$$\frac{4f^2}{3} - y^2 = mx$$

B

$$x = \frac{4f^2 - 3y^2}{3m}$$

C

$$\frac{4f^2}{3} = mx + y^2$$

D

Make m the subject of the formula

$$f = \sqrt{\frac{3(mx + y^2)}{4}}$$

E

$$x = \frac{4f^2}{3m} - \frac{y^2}{m}$$

F

$$4f^2 = 3(mx + y^2)$$

G

$$f^2 = \frac{3(mx + y^2)}{4}$$

Find the correct order for each set of steps.

Coordinate geometry (1)

A

$-4 = 6 + c$

B

The equation of the line is $y = 3x - 10$

C

Find the equation of the line parallel to $y = 3x - 7$ that goes through $(2, -4)$.

D

$c = -10$

E

Since $(2, -4)$ is on the line, $(2, -4)$ satisfies the equation.

$-4 = 3(2) + c$

F

$y = mx + c$

The gradient is 3 because the lines are parallel (ie $m_1 = m_2$).

$y = 3x + c$

Coordinate geometry (2)

A

$y = mx + c$

The gradient is 0.5

$y = 0.5\,x + c$

B

Since $(3, 4)$ is on the line, $(3, 4)$ satisfies the equation

$4 = 0.5\,(3) + c$

C

The equation of the line is

$y = 0.5\,x + 2.5$

D

$4 - 1.5 = c$

E

Find the equation of the line that has gradient 0.5 and goes through the point $(3, 4)$.

F

$c = 2.5$

Coordinate geometry

Put the instructions A – F into the correct order. For each instruction there is a matching example. List the example steps in the correct order.

A

Read the question.

B

Calculate the gradient.

Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

C

Choose one of the points in the question. Replace *x* and *y* by the numbers of the point.

D

Solve the equation for *c*.

E

Write the general equation of a line. Substitute the gradient you have calculated for *m*.

F

Write the equation of the line. Use the values of *m* and *c* that you have calculated.

1

$y = mx + c$

$m = \dfrac{7}{4},\ c = \dfrac{1}{2}$

$y = \dfrac{7}{4}x + \dfrac{1}{2}$

2

Choose (6, 11)

$11 = \dfrac{7}{4}(6) + c$

3

Find the equation of the line that goes through the points (2, 4) and (6, 11).

4

$m = \dfrac{11 - 4}{6 - 2}$

$= \dfrac{7}{4}$

5

$11 = \dfrac{42}{4} + c$

$c = \dfrac{1}{2}$

6

$y = mx + c$

$= \dfrac{7}{4}x + c$

Simultaneous equations (line/curve)

Problem 1

A

Use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve the equation for x. (It may be possible to solve the equation more simply by factorising or completing the square.)

B

Write the solutions as coordinates.

C

Read the question.

D

Substitute the linear expression into the equation of the curve.

E

Use one of the original equations of the question to solve for y. The linear equation is usually simplest to use for this. Substitute the known values of x. Do this once for each value of x.

F

Collect like terms. Collect all the terms on the same side of the equals sign.

G

Check the x and y pairs by substituting them into the other original equation.

H

Expand the brackets.

1

The solutions are (3, 4) and (–1.4, –4.8)

2

$x^2 + 4x^2 - 8x + 4 = 25$

3

$x^2 + (2x - 2)^2 = 25$

4

$y = 2x - 2$

when $x = 3$
$y = 6 - 2$
$= 4$

when $x = -1.4$
$y = -2.8 - 2$
$= -4.8$

5

$a = 5,\ b = -8,\ c = -21$

$x = \dfrac{8 \pm \sqrt{64 + 420}}{10}$

$= \dfrac{8 \pm 22}{10}$

$= 3, -1.4$

6

Find the points (x, y) which satisfy both equations:

$y = 2x - 2$

$x^2 + y^2 = 25$

7

$x^2 + y^2 = 25$

when $x = 3$, $y = 4$

$3^2 + 4^2 = 25$

when $x = -1.4$, $y = -4.8$

$(-1.4)^2 + (-4.8)^2 = 25$

8

$5x^2 - 8x - 21 = 0$

Put the instructions A – H into the correct order. For each instruction there are three corresponding examples, one for each problem. Put the steps of each problem into the correct order.

Problem 2

1

$y = 3x + 7$

when $x = -1$: $y = -3 + 7 = 4$

when $x = -2$: $y = -6 + 7 = 1$

2

$3x + 7 = x^2 + 6x + 9$

3

$a = 1,\ b = 3,\ c = 2$

$$x = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} = -1, -2$$

4

$y = (x + 3)^2$

$x = -1,\ y = 4$: $y = (-1 + 3)^2 = 4$

$x = -2,\ y = 1$: $y = (-2 + 3)^2 = 1$

5

The solutions are (–1,4) and (–2,1)

6

$x^2 + 3x + 2 = 0$

7

$3x + 7 = (x + 3)^2$

8

Solve these equations simultaneously:

$y = (x + 3)^2$

$y = 3x + 7$

Problem 3

1

$6x - x^2 + x - 6 = x + 2$

2

$$y = \frac{x+2}{x-1}$$

$x = 2,\ y = 4$: $y = \frac{4}{1} = 4$

$x = 4,\ y = 2$: $y = \frac{6}{3} = 2$

3

$-x^2 + 6x - 8 = 0$

4

$$6 - x = \frac{x+2}{x-1}$$

$(6 - x)(x - 1) = x + 2$

5

Find the points where the line $y = 6 - x$ intersects $y = \dfrac{x+2}{x-1}$

6

The solutions are (2, 4) and (4, 2).

7

$a = -1,\ b = 6,\ c = -8$

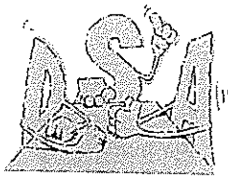
$$x = \frac{-6 \pm \sqrt{36-32}}{-2} = \frac{-6 \pm 2}{-2} = 2, 4$$

8

$y = 6 - x$

when $x = 2$: $y = 6 - 2 = 4$

when $x = 4$: $y = 6 - 4 = 2$



Standard normal distribution

Problem 1

A

Find what the question is asking for. This is usually near the end. Write it as a mathematical statement.

B

Write the answer as a sentence.

C

Sketch the distribution on a bell curve. Shade the region that is required to solve the problem.

D

Change the parameters and values to those of the standard normal distribution. This has a mean = 0 and standard deviation = 1.

E

Write the parameters of the distribution. For a normal distribution the parameters are the mean and the standard deviation.

F

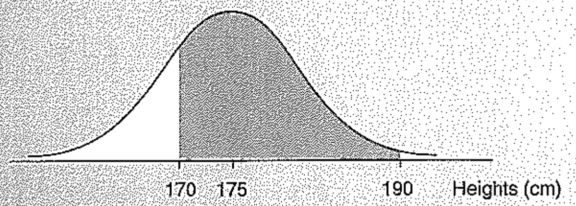
Look up the Z values in the tables. Use the symmetry of the curve to work out the probability. You may have to add or subtract.

G

Read the whole question.

1

X = heights of male security guards



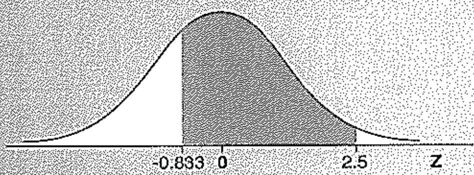
2

$$Z = \frac{X-\mu}{\sigma}$$

$P(170 \leqslant X \leqslant 190)$

$= P\left(\frac{170-175}{6} \leqslant Z \leqslant \frac{190-175}{6}\right)$

$= P(-0.833 \leqslant Z \leqslant 2.5)$



3

The probability that a male security guard chosen at random is between 170 and 190 cm tall is 0.7913.

4

Let X = the heights of male security guards.

$\mu = 175$ cm

$\sigma = 6$ cm

5

Security firms have found that the heights of their male guards are normally distributed with mean 175 cm and standard deviation 6 cm. Find the probability that a male security guard chosen at random is between 170 and 190 cm tall.

6

$P(-0.833 \leqslant Z \leqslant 2.5)$

$= 0.2975 + 0.4938$

$= 0.7913$

7

$P(170 \leqslant X \leqslant 190) = ?$

Put the instructions A–H into the correct order. For each instruction there are three examples, one for each problem. Put the steps of each problem into the correct order.

Problem 2

1 The probability that a bag of Yum Yum jellybeans weighs more than 210 grams is 0.0478.

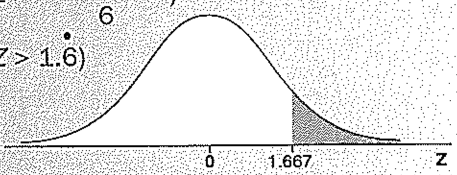
2 Quality control tests have shown that bags of Yum Yum jellybeans have weights that are normally distributed with mean 200 grams and standard deviation 6 grams. What is the probability that a bag weighs more than 210 grams?

3 $Z = \dfrac{X - \mu}{\sigma}$

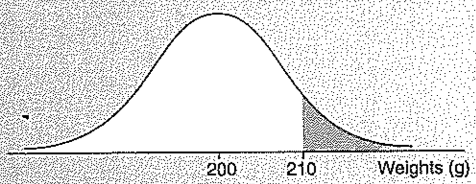
$P(X > 210)$

$= P(Z > \dfrac{210 - 200}{6})$

$= P(Z > 1.\dot{6})$



4 X = weights of bags of Yum Yum jellybeans.



5 Let X = the weights of bags of Yum Yum jellybeans.

$\mu = 200$ g
$\sigma = 6$ g

6 $= 0.5 - 0.4522$
$= 0.0478$

7 $P(X > 210) = ?$

Problem 3

1 Let X = the time taken to make bank uniforms.

$\mu = 720$ seconds
$\sigma = 32$ seconds

2 $= 0.4697 + 0.3248$
$= 0.7945$

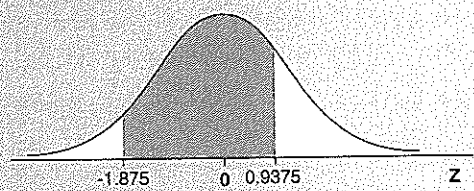
3 $P(660 < X < 750) = ?$

4 $Z = \dfrac{X - \mu}{\sigma}$

$P(660 < X < 750)$

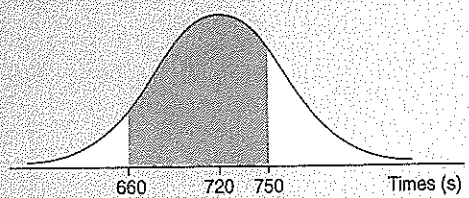
$= P(\dfrac{660 - 720}{32} < Z < \dfrac{750 - 720}{32})$

$= P(-1.875 < Z < 0.9375)$



5 It takes experienced machinists twelve minutes on average to make bank uniforms. The standard deviation is 32 seconds. What is the probability that a particular garment took between 11 and $12\frac{1}{2}$ minutes to make?

6 X = time taken to make bank uniforms.



7 The probability that a garment took between 11 and $12\frac{1}{2}$ minutes to make is 0.7945.

Optimisation

Problem 1

A

Draw a labelled diagram of the object.

B

The differentiated equation equals zero for an optimum solution.

Rearrange the differentiated equation in order to solve it.

C

Write an equation for the quantity to be optimised. This equation should start with a word, or a symbol that represents a word.

D

Look in the question for the known quantity. Use it and your diagram to write an equation. Simplify and rearrange the equation so that it has one variable as the subject.

E

Read the question.

F

Use the diagram to help answer the question.

Write the answer in a sentence.

G

Differentiate the quantity to be maximised or minimised.

H

Combine the two equations using substitution. There should only be two variables in the result.

1

Area = xy $\quad$ $y = 6 - x$

Area = $x(6 - x)$

$= 6x - x^2$

2

$$\frac{d\,(\text{Area})}{dx} = 6 - 2x$$

3

Ross bought some material and edging fringe in order to make a rectangular table cloth. What is the maximum possible area of the cloth if Ross has 12 metres of fringe to use?

4

$2x + 2y = 12$

$x + y = 6$

$y = 6 - x$

5

$0 = 6 - 2x$

$2x = 6$

$x = 3$

6

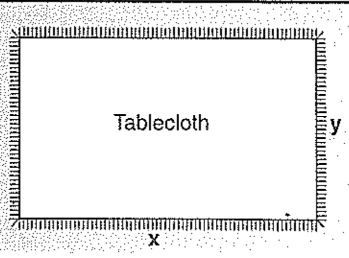
Area = xy

7

The maximum area of the table cloth is 9 m^2. This is achieved with dimensions 3 m by 3 m.

8

12 m of fringe



Put the instructions A-H into the correct order. For each instruction there are three examples, one for each problem. Put the steps of each problem into the correct order.

Problem 2

1

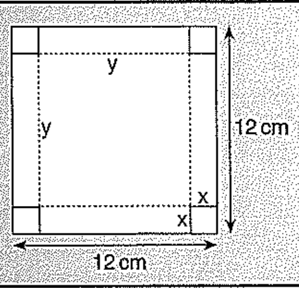
$0 = 12x^2 - 96x + 144$

$= x^2 - 8x + 12$

$= (x - 2)(x - 6)$

$x = 2, 6$

2



3

Volume $= xy^2$

4

Aroha is making open-topped boxes to hold the chocolates she sells. Each box is made from a 12 cm square piece of card. Small squares are cut from each corner and the sides folded up. What size should the cut squares be to maximise the volume of the boxes?

5

$y + 2x = 12$

$y = 12 - 2x$

6

$$\frac{d(\text{Volume})}{dx} = 12x^2 - 96x + 144$$

7

Volume $= xy^2 \quad y = 12 - 2x$

Volume $= x(12 - 2x)^2$

$= 4x^3 - 48x^2 + 144x$

8

The squares cut from the corners of the card should be 2 cm x 2 cm in order to maximise the volume. The maximum volume of the box is 128 cm^3.

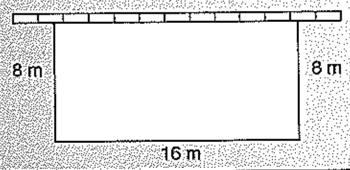
Problem 3

1

Length $= 2x + y$

2

The minimum length of border material is 32 m. This is for a garden 8 m x 16 m.

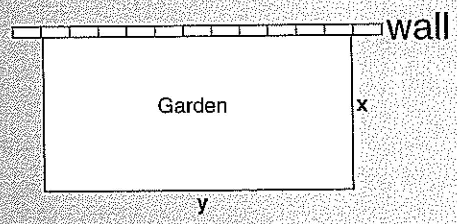


3

Length $= 2x + y \qquad y = \frac{128}{x}$

Length $= 2x + \frac{128}{x}$

4



5

$0 = 2 - \frac{128}{x^2}$

$2 = \frac{128}{x^2}$

$x = 8$ (In this question -8 is meaningless)

6

$xy = 128$ m^2

$y = \frac{128}{x}$

7

Jill is making a rectangular garden. She needs an area of 128 m^2. Jill wants to know the minimum length of border material needed to go around three sides of the garden. The fouth side is along a wall.

8

$$\frac{d(\text{Length})}{dx} = 2 - \frac{128}{x^2}$$

Trignometry rules

Problem 1

A

Use the values you have found to complete the problem by solving the formula.

B

Choose a formula.

C

Read the question.

D

Choose a trigonometric rule to calculate any other quantity that is needed.

E

Write your answer in a sentence.

F

Draw a labelled diagram.

G

Substitute the known values and solve the equation to calculate the value you need.

1

$= 0.05427$

$C = 86.89°$ (2dp)

2

Town planners are developing a paved area in the city. It is triangular with dimensions 12 m, 23.5 m and 25.8 m. What is the area of the ground they are paving?

3

$a = 12$ m

$b = 23.5$ m

$C = ?$

$$\text{Area} = \frac{1}{2}ab\sin C$$

4

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

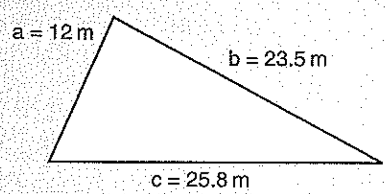
5

The area to be paved is 140.79 m^2 (2 dp)

6

$$\text{Area} = \frac{1}{2}ab\sin C$$
$$= 140.79\ m^2 \text{ (2dp)}$$

7



Put the instructions A – H into the correct order. For each instruction there are three corresponding examples, one for each problem. Put the steps of each problem into the correct order.

Problem 2

1

Stephanie is making triangular kites. She makes the pieces by measuring two angles and a length. If the pieces fit perfectly together, how much material is needed for 20 kites?

2

$$= \frac{3 \sin 53°}{\sin 100°}$$

$= 2.433$ m (3 dp)

3

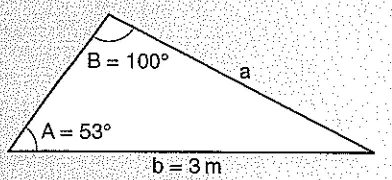
The area needed for 20 kites is $33.14\ m^2$ (2 dp).

4

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$a = \frac{b \sin A}{\sin B}$$

5



6

$a = ?$

$b = 3$ m

$C = 27°$

$$\text{Area} = \frac{1}{2} ab \sin C$$

7

$a = 2.433$ m

$b = 3$ m

$C = 27°$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$= 1.657\ m^2$ (3 dp)

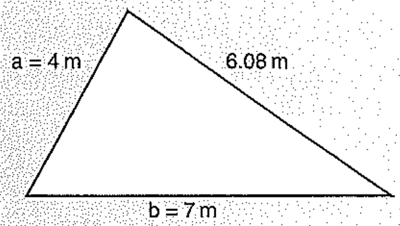
Problem 3

1

$= 0.5006$

$C = 60°$ (2 sf)

2



3

The area of the triangle is $12\ m^2$ (2 sf).

4

$a = 4$ m

$b = 7$ m

$C = ?$

$$\text{Area} = \frac{1}{2} ab \sin C$$

5

Calculate the area of a triangle that has sides of 4 m, 7 m and 6.08 m.

6

$a = 4$ m

$b = 7$ m

$C = 60°$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$= 12\ m^2$ (2 sf)

7

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

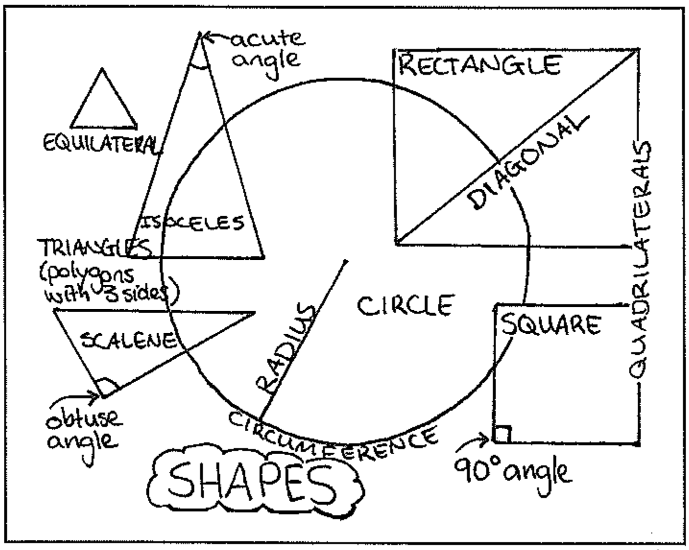
Language Mathematics Activities

These activities focus on the language aspect of mathematics. They aid understanding of the vocabulary used in different topics.

The word lists can be used in many ways. There are possibilities for creative written and oral communication, and the words can be used in groupwork or by individuals. The activities also provide opportunities for self evaluation by the learner and for tutor assessments.

Some ways of using the activities are listed below:

1 Display all the words from a topic list on one large sheet of paper. Group the words according to similar uses or meanings, and set it all out in a logical manner. Use arrows, diagrams, graphs, explanations etc. to help make the meanings of the words clear.
e.g.



2 Play a whole class game in two or more teams. Points are scored for each accurate description given. The teacher judges how many points each given description is worth:

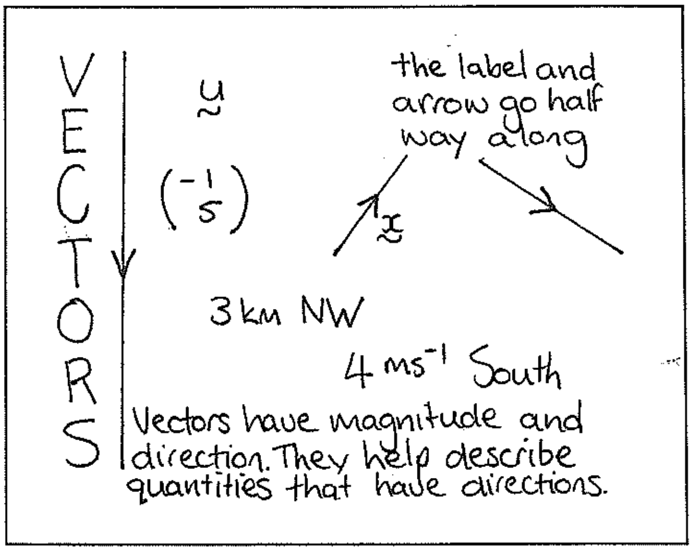
- 2 points for a full description
- 1 point for a partial description
- 0 points if no description is given.

Teams can take turns to choose which word they will define. Answers can be given by individuals, or alternatively, by the group working together.

3 Complete a written topic summary using the list as a starting point.

4 Make a poster to illustrate the meaning of one or more words from the list. Different students can complete one poster for each word as a class project.

e.g.



5 Prepare a short talk to explain the meaning and uses of one of the words, or a small group of words from the list.

Other language Mathematics Activities

- Research the history of the mathematical theory in a topic, for example probability theory, integration, set theory, trigonometry, statistical graphing, differential equations etc.
- Write an essay about a female or a male mathematician. Either choose a historical figure or someone currently working in a mathematical occupation. Write about the person and about the mathematics they use or used.
- Keep a mathematics journal. Express your feelings about mathematics, keep summaries of recent work learnt, interesting puzzles and investigations. Keep a record of mathematical articles from the newspaper.
- Write a discussion that might take place between the four mathematicians on the cover.
- Write a poem or a story about mathematics or with a mathematical theme.
- Write your own mathematical problems and swap with others to solve them.
- Write an essay about whether or not mathematics is a more important subject today than it has ever been. Include reasons that support your belief.

Circular measure

Radian

Arc

Radius

Centre

Circle

Revolution

Sector

Segment

180°

Area

Arc length

π

Area of a segment

$a = r\theta$

$A = \frac{1}{2} r^2\theta$

Degrees

Area of sector

Diagram

$A = \frac{1}{2} R^2 (\Theta - \text{SIN}\ \Theta)$

Trigonometry

For instructions, see page 49.

Sin θ

Angles

The sine rule

Tangent

Special triangles

Asymptote

Cosine θ

The cosine rule

Right angled triangles

$\cos^2 \theta + \sin^2 \theta = 1$

$y = A \sin Bx + C$

Tan θ

ϕ

Amplitude

Tables

Calculator

$A = \frac{1}{2} ab \sin C$

Period

2π

Shift up

Radians

$a^2 + b^2 = c^2$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

Area is half base x height

Applications

Shift to the right

$a^2 = b^2 + c^2 - 2bc \cos A$

Calculus

For instructions, see page 49.

Differentiate

Area under the curve

$\frac{dy}{dx}$

Definite integral

Upper bound

Anti-differentiate

Gradient function

Equation of the tangent

Lower bound

Indefinite integral

$\frac{d^2y}{dx^2}$

Velocity

Minimise

Displacement

Acceleration

Maximise

y'

Integrate with respect to x

y''

$f(x)$

Rate of change

Double derivative

$f'(x)$

$f''(x)$

Limits

The slope of the normal at a point

Coodinate geometry

For instructions, see page 49.

Distance between two points

Parallel lines

Mediator

Gradient

y-intercept

$m_1\, m_2 = -1$

Origin

Distance formula

x-axis

Coordinates

Cartesian plane

Equation of a line

$m_1 = m_2$

Midpoint

Set of axes

x-intercept

Perpendicular lines

Slope

y-axis

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\sqrt{(x_2 - x_1{}^2) + (y_2 - y_1)^2}$$

$ax + by + c = 0$

Coordinate plane

$y = mx + c$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$